Tutorial 4

Resilient Communication Systems
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MIMO Detectors I

RESILIENT COMMUNICATION SYSTEMS

Problem 1

Sphere decoding: Consider a MIMO communication system, $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$, with the following channel matrix

$$\mathbf{H} = \begin{bmatrix} 1 & 0.2 + 0.4j \\ 0.5 + 0.1j & 1 \end{bmatrix}.$$

Let $\mathbf{x}_n \in \mathcal{A}$ is taken from QAM constellation, i.e., $\mathcal{A} = \pm 1 \pm j$. Assuming the received signal as $\mathbf{y} = [1.5 - 0.6j, 2.9 + 1.05j]^{\mathsf{T}}$, retrieve the transmitted signal using the sphere decoding algorithm.

1. **Step 1:** Firstly, derive QL decomposition of matrix $\mathbf{H} = \mathbf{Q}\mathbf{L}$, then compute $\hat{\mathbf{y}} = \mathbf{Q}^{\mathsf{H}}\mathbf{y}$.
2. **Step 2:** Compute $f_l(\mathbf{x}_l) = \|\hat{y}_l - \sum_{i=1}^{l} [\mathbf{L}]_{l,i} x_i\|^2$ for any required (i, l) pair.

Problem 2

Derive the diversity gain of Maximum-likelihood (ML) detection [1].

1. **Step 1:** Firstly, consider worst-case pairwise error probability (PEP) ($\Pr\{\mathbf{x}_i \to \mathbf{x}_j | \mathbf{H}\}$).
2. **Step 2:** Compute the diversity gain with the following definition: $G_d = \lim_{\gamma \to \infty} -\frac{\log(P_e(\gamma))}{\log(\gamma)}$ where γ is SNR.

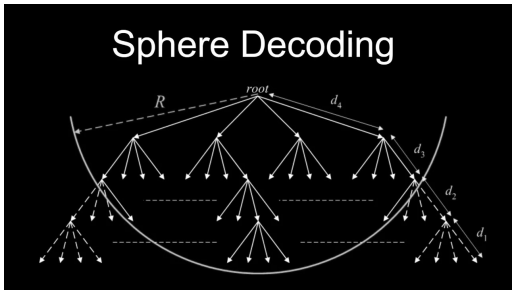
Hint 1: In PEP, it is assumed that $\mathbf{x}_i$ was transmitted and $\mathbf{x}_i$ and $\mathbf{x}_j$ are the only codewords in the codebook.
Hint 2: $\mathbb{E}\left\{Q\left(\sqrt{2\|\mathbf{h}\|^2\gamma}\right)\right\} \approx \binom{2N-1}{N}\left(\frac{1}{4\gamma}\right)^N$, where $\gamma \to +\infty$, $[\mathbf{h}]_i \sim \mathcal{CN}(0,1), \forall i$ and i.i.d., and, Q is Q-function.

References

[1] L. Zheng and D. N. C. Tse, "Diversity and multiplexing: A fundamental tradeoff in multiple-antenna channels," *IEEE Transactions on information theory*, vol. 49, no. 5, pp. 1073–1096, 2003.

[2] U. Fincke and M. Pohst, "Improved methods for calculating vectors of short length in a lattice, including a complexity analysis," *Mathematics of Computation*, vol. 44, no. 170, p. 463–471, 1985.

Sphere decoding is an algorithmic technique used to solve the integer least squares problem, which involves finding the closest lattice point to a given vector. In the context of MIMO communication systems, sphere decoding is employed to detect transmitted symbols by efficiently searching for the maximum likelihood solution within a constrained search space. By limiting the search to lattice points within a hypersphere of a certain radius, the algorithm significantly reduces computational complexity compared to exhaustive search methods, especially in high-dimensional scenarios. This approach enables near-optimal detection performance with manageable computational resources, making it a valuable tool in modern digital communication systems. The following Youtube video explains briefly Sphere decoding:
Sphere decoding



Sphere decoding

RESILIENT COMMUNICATION SYSTEMS

Michael Pohst

Michael Pohst is a German mathematician renowned for his contributions to computational number theory and algebra. He co-authored the influential book Algorithmic Algebraic Number Theory with Hans Zassenhaus, which provides a comprehensive introduction to constructive algebraic number theory and has been a valuable resource for both students and researchers in the field. In 1985, Pohst, along with U. Fincke, introduced the sphere decoding algorithm in their paper "Improved Methods for Calculating Vectors of Short Length in a Lattice, Including a Complexity Analysis" [2]. This algorithm has become a cornerstone in MIMO detection, offering near-optimal performance with reduced computational complexity. Throughout his career, Pohst has made significant contributions to the development of algorithms for algebraic number theory, including methods for computing class groups and unit groups of algebraic number fields. His work has had a lasting impact on both theoretical research and practical applications in areas such as coding theory and cryptography. Read more in Wikipedia.


Michael Pohst
Born: 5 June 1945
age: 79

Reference: Pohst, Michael, and Hans Zassenhaus. Algorithmic algebraic number theory. Vol. 30. Cambridge University Press, 1997.